Tutorial in bondage-physics: Timing of your ice-release system

I am a girl, but I am also one of these weirdos who hold a PhD in physics. My little program works with the CD tray release. But this release is somewhat unreliable in the worst case (Hacker assault etc.). As a back-up I always use the ice-release method, because ice will always melt. But it is somewhat difficult to time it. But as I am a physicist I just did some little calculations – and here you go: I present you some formulas that allow everybody an almost exact timing of their ice-release system.
For all the weirdo nerds amongst you: the key is the Stefan-Boltzmann radiation law.
But nothing to worry about for everybody else. The results are very easy formulas for everybody to apply with an ordinary calculator and a measure tape. Enjoy!

I now will take you at my hands now and lead you through. I assume that your are perfectly alright with that I do not molest you with all these awkward integrals and algebraic derivations that have to be done and to just present you the necessary results, right? For the nosy nerds amongst you I will make some comments on how to do it.

Physics works with the metrical system. So for all of you who do not use the metrical system in your all-days life, I shall help you on the road:

Inch to metres: 1 inch = 0.025 m, or 1 m = 40 inch

Feet to metres: 1 ft = 0.31 m, or 1 m = 3.22 ft

Pound to kilograms: 1 lb = 0.45 kg, or 1 kg = 2.22 lb

Pinte(US) to liters: 1P = 0.47 L, or 1L = 2.13 P(US)

Pinte(UK) to liters: 1P = 0.57 L, or 1 L = 1.75 P(UK)

In the metrical system we have: **1 kg = 1L! So when you convert your pinte into liter, you already have your kilograms!**

Surface-Area

The surface-area of the bin you use is an extremely important quantity. As I do not know the measures of the bin you use, I cannot spare to show you how to calculate the surface-area. You are advised to use a cylindrical bin, because the temperature field inside is approximately a well-behaving one.

Cylindrical bin. You need:

Geometrical constant **π** = 3.14

Diameter of your bin's floor or top in **metres**. We call it **d** now.

Height of your bin in **metres**. We call it **h** now.

The surface-area is then: **A = π*d*h + 2*π*(d/2)^2** (^2 means the second power)

The bin-material to be used is preferably glass or thin material as paperboard with an aluminium layer on the inside, or aluminium! Pure aluminium but is the Speedy Gonzalez of heat transport. Be prepared that the times given below reduce appreciably.
Your bin's walls should not be thicker than around 2 millimetres = 0.002 m

How to do the timing:

Surely you have an idea for how long you want to stay in your bondage. So you know the **time t(total)** in advance. Your question is how much water you will have to use. I will now show you how you have to do this. Please note that all times below are given in **hours**.

The melting process involves two steps. The first step is the heating up of your bin from the temperature of the freezer **T(freezer) = 255 K = -18°C = 0°F** to the melting temperature **T(melt) =273 K = 0°C = 32°F**.

t(heat) = (m / A) * 0.056, m(mass), A(surface-area).
(for the nosy ones: take the heat capacity of ice, calculate the differential flux of heat(temperature as a function of time) and equate it with the S-B-law, separate the variables, integrate about the temperature interval and the time, divide by 3600 to get hours. Result: 0.056 h*m^2/kg. Background field at T = 300 K. Integrate numerically, because else you get some awkward hypergeometric functions.)
The time **t(heat)** is the time that is needed to heat up the bin to the melting temperature. This is done by the thermal radiation field in your room, it comes free house.

The second step is the melting process itself. The temperature of the bin will stay at **T(melt)** until most of the ice has melted into water. Until this has not happened, there will be a mixture of water and ice. But when there is more water than ice, the melting will take up a little speed. The time to melt down all the ice into water is given by

t(melt) = (m / A) * 0.62, m(mass), A(surface area).
(for the nosy ones: take the melting energy of ice, divide by S-B-law, background field at T = 300 K., divide by 3600 to get hours. Result: 0.62 h*m^2/kg. The temperature remains constant during the melting process because this is a phase-transition.)
The melting is again done by the thermal radiation field, free house.

When you compare the coefficients of **t(heat)** and **t(melt)**, you understand that the melting process takes much longer than the heating process. It can be very unpleasant to misjudge the melting time **t(melt)**.

The total time is **t(total) = 0.5*(t(heat) + t(melt)) = 0.34*(m/A).**
It follows from my experience that the total time equals the average time. The average stems from a give or take a little treatment of the conductivity of heat.
(for the nosy ones: equate the S-B-law with the law of heat conduction. For reasonably small temperature gradients on the shell-surface of the bin the factor of difference is around 2. I was surprised too. Background-field at T = 300 K.)

Example
As you are bright-minded (no doubt about that) you do a little play with all these above numbers and make yourself a table, in order to gain a better judgment of what you want – and what you definitely do not want.
Again: you MUST use metrical units! Else you get it utterly wrong!
1 m = 40 inches!
I will do the following calculations with the measures of one of my cylindrical bins:

Diameter d = 2.4 inch/40 = 0.06 m, maximum Height h = 3.4 inch/40 = 0.085 m

The surface-area thus is $A(h) = \pi*d*h + 2*\pi*(d/2)^2 = 0.022\ m^2$ for the maximum height h = 0.085 m. A(h) means that the surface-area is a function of h. For each height h you have to calculate it anew.

We can easily express the mass of the water that is used by the volume (density of water is $1\ kg/m^3$) of the cylinder:

$V(h) = 1000*\pi*(d/2)^2*h$. Thus V(h) = 0.24 kg, when h = 0.085 m. This is the maximum amount of water that can be used in this bin. Again V(h) is the volume as a function of the height h. For each height you have to calculate it anew.

The formula for the heat up time becomes t(heat) = 0.056*V(h)/A(h).

The formula for the melting time becomes t(melt) = 0.62*V(h)/A(h).

t(total) = 0.5*(0.056+0.62)*(V(h)/A(h)) = 0.34*(V(h)/A(h))!
I am not providing you with the inverse of this formula. For the non nerd ones it may look a little, say, scary. And the nerds can do it on their own anyway.

The advantage of these formulas is now, that they do only depend on the height h of the water you have filled in. Thus it is easy to measure what you want to have. Remember that 0.085 m = 8.5 cm = 40*0.085 m = 3.4 inch. This is very easy to measure.

By applying our formulas we finally obtain our sojourn time in bondage:

h	0.085 m	0.075 m	0.065 m	0.055 m	0.045 m	0.035 m	0.025 m
t(heat)	0.62 h	0.60 h	0.58 h	0.55 h	0.51 h	0.45 h	0.38 h
t(melt)	6.84 h	6.61 h	6.33 h	6.0 h	5.55 h	4.9 h	4.21 h
t(total)	3.73 h	3.6 h	3.45 h	3.3 h	3.03 h	2.7 h	2.3 h

Times in hours! For minutes multiply with 60 (e.g. 3.73 h = 3 h 44 min).
(for the nosy ones: the decay of the time is not linear, because the top and bottom surfaces do not change with different heights.)
Also: the accuracy declines with the height, because with only 0.025 m height of water your key will be free very much earlier than the whole process will take. To avoid this, chose another bin or enhance the height of the water.

The times given are upper bounds. There are additional heat sources:
heating, sunlight through the window,
your bin right in the sunlight, stupid one,
etc., etc,...............
This are only tiny effects, but they will speed up the melting process a little bit.

Also, all above only works as said when there is absolutely no possibility to draw your key out before all the ice has melted. As this is mostly not the case, you can always use a little more water.

I advise you, after you have chosen your favorite bin, that you make yourself a table as the one above. Then you can read off instantly what you need in order to carry out your fantasy. Do some experiments in order to feel safe and easy.

Happy ice release! And never ever again you dare to say that physics is only some crazy remote weirdo's stuff! It's fun, isn't it?(-: